

Semiclassical transport in nearly symmetric quantum dots. II. Symmetry breaking due to asymmetric leads

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In this work—the second of a pair of articles—we consider transport through spatially symmetric quantum dots with leads whose widths or positions do not obey the spatial symmetry. We use the semiclassical theory of transport to find the symmetry-induced contributions to weak localization corrections and universal conductance fluctuations for dots with left-right, up-down, inversion, and fourfold symmetries. We show that all these contributions are suppressed by asymmetric leads; however, they remain finite whenever leads intersect with their images under the symmetry operation. For an up-down symmetric dot, this means that the contributions can be finite even if one of the leads is completely asymmetric. We find that the suppression of the contributions to universal conductance fluctuations is the square of the suppression of contributions to weak localization. Finally, we develop a random-matrix theory model which enables us to numerically confirm these results.

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I. INTRODUCTION

This work is the second of a pair of articles on mesoscopic transport through chaotic quantum dots with spatial symmetries (see Ref. [1] for part I). In both works we use recent advances in semiclassical techniques to address the effect of spatial symmetries on weak localization (WL) corrections and universal conductance fluctuations (UCFs). The aim of the first article was to identify the microscopic origin of properties that were earlier only known from phenomenological random-matrix theory (RMT) [2–5], and furthermore to extend the considerations to situations in which RMT is not easily applicable. In particular, this includes scenarios where symmetries are only partially preserved. To this end, the first article [1] also considered the combined effects of magnetic fields, a finite Ehrenfest time, and dephasing on symmetric systems and also discussed the reduction of symmetry-related interference effects by deformations of the dots.

In the present paper, we contrast this “internal” symmetry breaking with symmetry breaking which is due to the position or shape of the leads (for examples of such situations see Fig. 1). We ask what happens to the transport if we take a symmetric dot coupled to leads which respect the symmetry and then start moving one of the leads. In the fully symmetric situation, the magnitude of UCFs is doubled for each independent symmetry, while the weak localization correction can be either increased or reduced (sometimes remain unaffected) depending on the spatial symmetry in question [1–3]. Are these symmetry-induced effects modified when the leads are deformed or displaced? If so, are they sensitive to displacement on a quantum scale (of order of a Fermi wavelength) or a classical scale (of order of a lead width)?

The present literature does not offer much guidance to answer these questions—indeed, the knowledge on transport in spatially symmetric systems with displaced leads is rather limited. Reference [6] reports that the distribution of transmission eigenvalues of a left-right symmetric dot with completely asymmetrically placed leads differs slightly from the

distribution of a completely asymmetric dot. Because the difference is small, symmetric systems (such as stadium billiards) with displaced leads are indeed often used as representatives of completely asymmetric systems (see, e.g., Refs. [4,5]). Recent works of one of the authors, on the other hand, identify a huge conductance peak in weakly coupled mirror-symmetric double dots which still remains large even when the leads are not placed symmetrically [7].

A simple consideration of weak localization quickly convinces us that it could never be as robust as the above-mentioned huge conductance peak in double dots. In systems without spatial symmetries, weak localization is the counterpart of coherent backscattering—particle conservation guarantees that you cannot have one without the other. Systems with spatial symmetries have additional coherent back- and forward-scattering contributions (as discussed in the first of this pair of articles [1]). These contributions rely on interference between paths that are related by spatial symmetry. If those paths do not both couple to the leads, they cannot generate an interference contribution to conductance. Thus, if we displace one lead so much that there is no intersection with its spatially symmetric partner ($W_{\cap}=0$ in Fig. 1) then the contributions to coherent forward scattering due to the spatial symmetries must vanish.

The precise distance by which one has to move the lead to substantially suppress the symmetry-related contributions depends on the detailed position dependence of the coherent forward- and backscattering peaks. In principle, these coherent interference patterns could oscillate on a scale of a wavelength, and thus one might imagine that a small displacement of that order would suffice. The calculations and numerical computations presented by us here show that this is not the case. Instead, the coherent forward- and backscattering peaks have a width of order the lead width and do not oscillate on the scale of a wavelength.

These considerations entail that the displacement of leads in internally symmetric systems offers a unique means to study coherent forward- and backscattering processes. From photonic systems it is known that the shape of the coherent

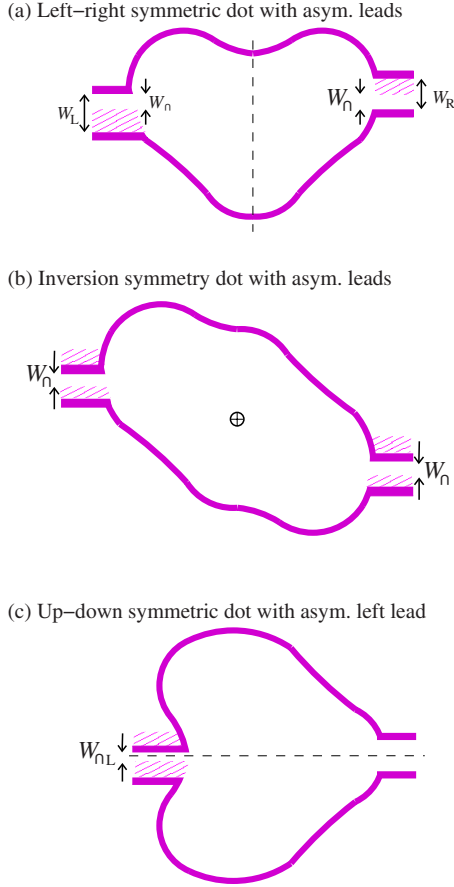


FIG. 1. (Color online) (a) A quantum dot with a left-right mirror symmetry coupled to leads which do not respect that symmetry. The left lead (L) has width W_L ; the right lead (R) has width W_R . The intersection between lead L and the mirror image of lead R has width W_n . If the L and R leads have no intersection under the mirror symmetry then $W_n=0$. (b) Same for a quantum dot with inversion symmetry. (c) A quantum dot with up-down symmetry for which each symmetry-respecting lead is mapped onto itself. In the figure, the left lead is displaced, which reduces the intersection W_{nL} of this lead with its mirror image.

backscattering cone provides valuable information on the multiple scattering in a sample [8,9]. Based on the results of the present work, transport measurements with gradually displaced leads promise to give similar insight into the dynamics of electronic systems.

This work is organized as follows. Section II introduces notation and provides a condensed review of the basic semiclassical concepts elaborated in more detail in the first of this pair of articles [1]. The following sections describe the consequences of displaced leads for the weak localization correction in systems with left-right symmetry (Sec. III), inversion symmetry (Sec. IV), up-down symmetry (Sec. V), and fourfold symmetry (Sec. VI). In Sec. VII we study the magnitude of universal conductance fluctuations for all types of symmetry. Finally, in Sec. VIII we generalize the phenomenological RMT model of symmetry breaking (presented in [1]) to the case of displaced leads and compare the results of numerical computations to the semiclassical predictions. Our conclusions are collected in Sec. IX. The Appendix contains

some further details on the semiclassical calculation of universal conductance fluctuations.

II. BACKGROUND

To make this paper self-contained we here first fix notation and then briefly summarize the main concepts of the theory of semiclassical transport in systems with spatial symmetries developed in the first of this pair of articles [1].

A. Characteristic scales

We consider chaotic quantum dots of size L [area $A = \mathcal{O}(L^2)$ and circumference $C = \mathcal{O}(L)$] which may possess any of the following three types of spatial symmetry: a left-right mirror symmetry, an inversion symmetry, and an up-down mirror symmetry. We also consider fourfold symmetric systems which simultaneously possess all the above symmetries. The quantum dot is perfectly coupled to two leads, labeled left (L) and right (R) and carrying N_L and N_R modes, where $N_\kappa = p_F W_\kappa / (\pi \hbar) \gg 1$ for $\kappa \in L, R$ (here p_F is the Fermi momentum; we also denote the Fermi velocity by v_F). The quantum dynamics in the dot is characterized by a number of time scales given by the time of flight $\tau_0 = \pi A / C v_F$ between successive reflections off the boundaries, the dwell time $\tau_D = \tau_0 C / (W_L + W_R)$, the dephasing time $\tau_\phi = 1 / \gamma_\phi$ (where γ_ϕ is the dephasing rate), and a time scale $\tau_B = (B_0 / B)^2 \tau_0$ on which a magnetic field destroys time-reversal symmetry. Here, $B_0 \sim h / (eA)$ is a characteristic field strength at which about one flux quantum penetrates the quantum dot. In transport, the effect of a magnetic field is felt at a smaller magnetic field

$$B_c = a B_0 \sqrt{\tau_0 / 2 \tau_D}, \quad (1)$$

where a is a system-specific parameter of order 1 [10]. Furthermore, the quantum-to-classical crossover is characterized by the open-system Ehrenfest time $\tau_E^o = \Lambda^{-1} \ln[W^2 / (L \lambda_F)]$ and the closed-system Ehrenfest time $\tau_E^c = \Lambda^{-1} \ln[L / \lambda_F]$, where Λ is the classical Lyapunov exponent and λ_F is the Fermi wavelength [11].

In contrast to Ref. [1], we here consider the possibility that the leads do not respect the symmetry of the dot. As shown in Fig. 1, the displacement from the symmetry-respecting position is characterized by the overlap of leads under the relevant symmetry operation. For left-right mirror symmetry and inversion symmetry, this is the width W_n of the intersection of a lead with the image of the other lead. An up-down symmetry maps each symmetry-respecting lead onto itself. The displacement of lead L (R) is then characterized by the width W_{nL} (W_{nR}) of the intersection of this lead with its own mirror image. In a fourfold symmetric system, the displacement is characterized by the various widths of intersections with respect to the individual symmetries (W_{nLR} for left-right mirror symmetry, W_{ninv} for inversion symmetry, $W_{nUD:L}$ for up-down mirror symmetry of lead L, and $W_{nUD:R}$ for up-down mirror symmetry of lead R).

B. Semiclassical theory of transport

The semiclassical theory of transport [12,13] expresses the transport through a quantum dot in terms of classical

paths γ, γ' which connect point y_0 on lead L to point y on lead R. Summing over lead modes as in Ref. [14], the dimensionless conductance (conductance in units of $2e^2/h$) is given by

$$g = \frac{1}{2\pi\hbar} \int_L dy_0 \int_R dy \sum_{\gamma, \gamma'} A_\gamma A_{\gamma'} e^{i(S_\gamma - S_{\gamma'})/\hbar}, \quad (2)$$

where $S_\gamma = \int_\gamma \mathbf{p} d\mathbf{r}$ denotes the classical action of a path and the amplitude A_γ is related to the square root of the path's stability.

For most pairs of γ and γ' the exponential in Eq. (2) oscillates wildly as one changes the energy or the dot shape. Thus, they make no contribution to the average conductance (where one averages over energy, dot shape, or both). The contributions that survive averaging are those where the pairs of paths have similar actions $S_\gamma \approx S_{\gamma'}$ for a broad range of energies and dot shapes. In particular, this is the case for the ‘‘diagonal contributions’’ to the above double sum (with $\gamma' = \gamma$), which can be analyzed using the sum rule (in the spirit of Eq. (B6) of Ref. [12])

$$\begin{aligned} \sum_\gamma A_\gamma^2[\dots]_\gamma &= \int_{-\pi/2}^{\pi/2} d\theta_0 \int_{-\pi/2}^{\pi/2} d\theta_F \cos \theta_0 \\ &\times \int_0^\infty dt \tilde{P}(\mathbf{Y}, \mathbf{Y}_0; t)[\dots]_{\mathbf{Y}_0}. \end{aligned} \quad (3)$$

Here we define $\tilde{P}(\mathbf{Y}, \mathbf{Y}_0; t) \delta y \delta \theta \delta t$ as the classical probability for a particle to go from an initial position and momentum angle of $\mathbf{Y}_0 \equiv (y_0, \theta_0)$ on lead L to within $(\delta y, \delta \theta)$ of $\mathbf{Y} = (y, \theta)$ on lead R in a time within δt of t . The average of \tilde{P} over an ensemble of dots or over energy results in a smooth function. If the dynamics are mixing on a time scale $\ll \tau_D$, one can approximate $\langle \tilde{P}(\mathbf{Y}; \mathbf{Y}_0; t) \rangle = e^{-t/\tau_D} \cos \theta / [2(W_L + W_R)\tau_D]$, which results in the classical Drude conductance

$$\langle g \rangle_D = \frac{N_L N_R}{(N_L + N_R)}. \quad (4)$$

Quantum corrections to this result originate from correlations of paths γ and γ' which are not identical but closely related by additional discrete symmetries in the system. For asymmetric quantum dots the only possible additional symmetry is time-reversal symmetry, which results in the ordinary weak localization correction [15–17] and associated coherent-backscattering peak [14, 16, 18] for systems whose classical dynamics exhibit hyperbolic chaos. The identification of possible pairings is also at the heart of the calculation of the magnitude $\text{var}(g)$ of universal conductance fluctuations, which in the semiclassical theory naturally takes the form of a quadruple sum over classical paths [19, 20].

Spatial symmetries in such chaotic systems induce further possible pairings both for the average conductance as well as for its variance, which are discussed in detail in the first article in this series [1]. In the following sections we revisit these results and extend them to the case of displaced leads, which is far richer than the case of symmetry-respecting leads.

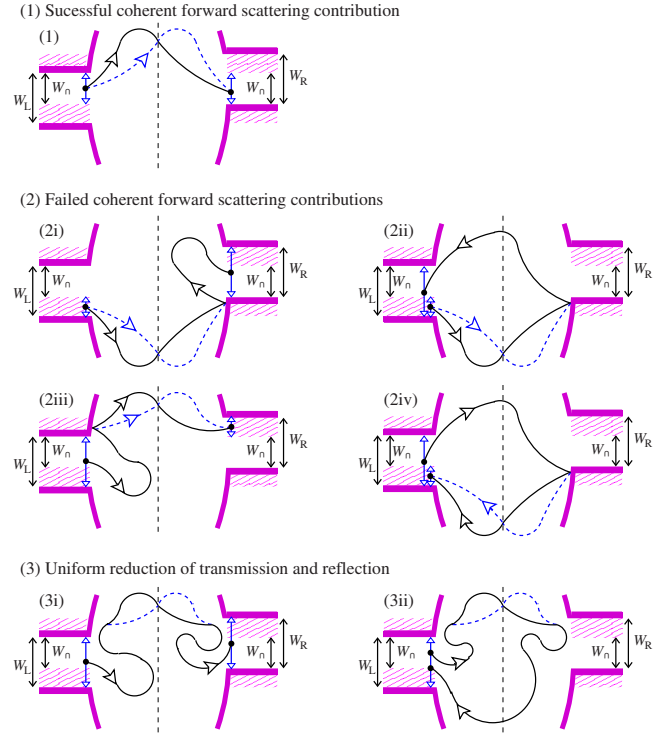


FIG. 2. (Color online) List of interference contributions to the conductance for a dot with left-right mirror symmetry when the leads are *asymmetric*. Here the leads have widths W_L and W_R and are centered at different places. The intersection of the L lead and the R lead’s mirror image has a width W_\cap and is indicated by the unshaded part of the L lead. The sketches on the left are all contributions to transmission from the L lead to the R lead (hence, the contributions to conductance). The sketches on the right are all contributions to reflection from the L lead back to the L lead.

III. LEFT-RIGHT SYMMETRIC QUANTUM DOT WITH DISPLACED LEADS

We first consider a left-right mirror-symmetric system with leads that are (partially or fully) displaced from the symmetry-respecting configuration. As shown in Fig. 1(a), the leads are of different widths and centered at different places. The amount of symmetry breaking is characterized by the (possibly vanishing) width W_\cap of intersection between lead L and the mirror image of lead R. In Fig. 2 we show the path pairings for all symmetry-induced interference corrections to the average conductance. (There is a strong resemblance between these contributions and the weak localization correction for systems with leads that contain tunnel barriers; in particular compare the *failed coherent forward scattering* contributions in Fig. 2 of this paper with the *failed coherent backscattering* contributions in Fig. 4 of Ref. [21]). None of the contributions listed in Fig. 2 are particularly difficult to calculate using the method presented in the first of this pair of articles [1]. This method involves folding paths under the spatial symmetry to find ways in which one can construct pairings between paths or their images with pairings switching at ‘‘effective’’ encounters; see Fig. 3. The difficulty is to find all contributions. One crucial check is to verify that the sum of all interference contributions to trans-

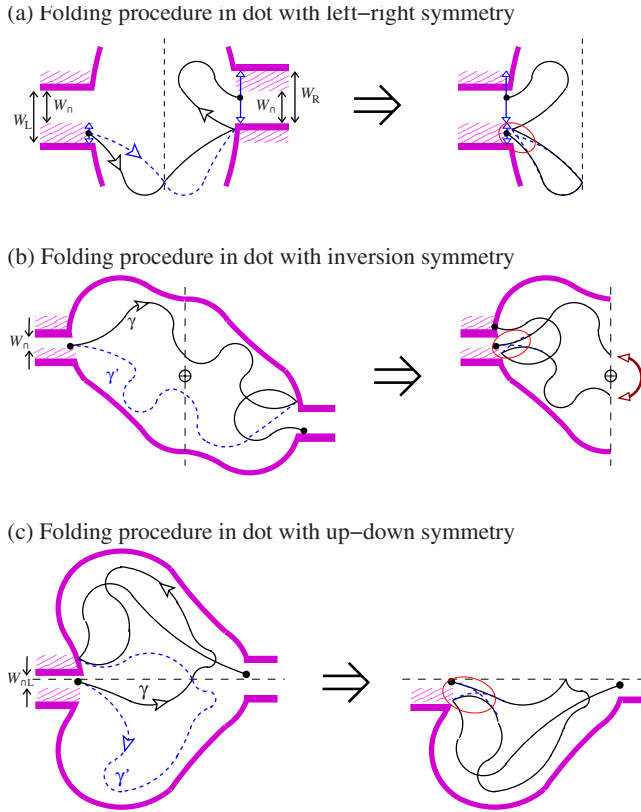


FIG. 3. (Color online) To find the nontrivial path pairings and to evaluate the phase difference between the paths, we use the folding procedure introduced in Ref. [1] (for symmetric leads). Here we consider the extra contributions generated by the fact the leads are asymmetric (i.e., contributions 2i–iv in Fig. 2). For each spatial symmetry, we give one example of the folding procedure for an unsuccessful coherent forward-scattering (or backscattering). The ellipses mark the effective encounters, where paths interchange their pairing. The other contributions are easily analyzed in the same way.

mission and reflection gives zero thereby ensuring particle conservation.

The main difference from the equivalent calculation for a system with symmetric leads (cf. Ref. [1]) is that here a pair of symmetry-related paths has a shorter *joint* survival time than the pairs of identical paths in the diagonal contribution. When the leads are symmetrically placed, the probability of a path staying in the dot (not hitting a lead) is strictly identical to the probability of its mirror image staying in the dot. This ceases to be the case when the leads are not symmetric. We deal with this by explicitly considering all situations where a path hits a lead (in which case it escapes from the system) or the mirror image of a lead (in which its mirror image will escape from the system). The probability that either of the processes occurs is $(W_L + W_R - W_\cap)/C$ per bounce at the boundary of the dot, where C is the circumference of the dot. We therefore define a modified dwell time

$$\tau'_D = \tau_D \frac{W_L + W_R}{2(W_L + W_R - W_\cap)} = \tau_D \frac{N_L + N_R}{2(N_L + N_R - N_\cap)}, \quad (5)$$

which characterizes the probability $\exp[-t/\tau'_D]$ that a path *and* its mirror image are *both* still in the dot at time t . We use this probability in place of $\exp[-t/\tau_D]$ in evaluating all parts of contributions 1 and 2 in Fig. 2 where the paths are the mirror image of each other.

A. Successful and failed forward-scattering contributions

The contribution of paths of the type labeled 1 and 2i–iv in Fig. 2 have an effective encounter close to a lead. These contributions are similar to certain contributions in an asymmetric system with tunnel barriers [21] and, hence, we use a similar method to analyze them here. The behavior of path γ' is completely determined by that of path γ so the two paths have the same amplitudes, $A_{\gamma'} = A_\gamma$. The action difference between them is $(S_\gamma - S_{\gamma'}) = (p_{0\perp} + m\Lambda r_{0\perp})r_{0\perp}$, where $(r_{0\perp}, p_{0\perp})$ is the component of $(\mathbf{Y} - \mathbf{Y}_0)$ which is perpendicular to the direction of path γ at \mathbf{Y} [22]. Using the sum rule in Eq. (3), we see that the contribution 1 in Fig. 2 is given by (cf. contribution LR:a in Ref. [1])

$$\langle \delta g \rangle_{\text{LR};1} = (2\pi\hbar)^{-2} \int_\cap d\mathbf{Y}_0 \int_\cap d\mathbf{Y} \int_0^\infty dt p_F \times \cos \theta_0 \langle P'(\mathbf{Y}, \mathbf{Y}_0; t) \rangle \text{Re}[e^{i(S_\gamma - S_{\gamma'})/\hbar}]. \quad (6)$$

The limits on the integral indicate that we only integrate over the region of the leads which have an overlap with each other under the left-right mirror symmetry (the regions of width W_\cap marked in Fig. 2).

The survival probability $\langle P'(\mathbf{Y}, \mathbf{Y}_0; t) \rangle = \exp[-t/\tau'_D] \delta r \delta \theta / [\pi(W_L + W_R - W_\cap)\tau'_D]$ is that of a path and its mirror image. The probability per unit time for path γ to hit within $(\delta r, \delta \theta)$ of a given point in the region of phase space defined by the union of leads and their mirror images is $\langle P'(\mathbf{Y}, \mathbf{Y}_0; t) \rangle \delta \mathbf{Y}$ where $\delta \mathbf{Y} \equiv \delta r \delta \theta$. Note that it is τ'_D rather than τ_D which gives the decay rate of $\langle P'(\mathbf{Y}, \mathbf{Y}_0; t) \rangle$. We express the \mathbf{Y}_0 integral in terms of the relative coordinates $(r_{0\perp}, p_{0\perp})$ and define $T'_W(r_{0\perp}, p_{0\perp})$ and $T'_L(r_{0\perp}, p_{0\perp})$ as the time between touching the lead and the perpendicular distance between γ and γ' becoming of order W and L , respectively. For times less than $T'_W(r_{0\perp}, p_{0\perp})$, two path segments are closer than a lead width to each other, and are almost mirror images of the other two (which are also closer than a lead width to each other). Thus their joint survival probability is the survival probability of a path and its mirror image. For times longer than this the path pairs escape independently, but since the pairs are made of a path and its mirror image, the escape rate is τ'_D not τ_D . The t integral in Eq. (6) must have a lower cutoff at $2T'_L(r_{0\perp}, p_{0\perp})$ because that is the minimum time for reconvergence. (For shorter times there is no contribution, because path γ and γ' must separate to a distance of order the dot size if they are going to reconverge at the other lead). Thus, we have

$$\int_\cap d\mathbf{Y} \int_0^\infty dt \langle P'(\mathbf{Y}, \mathbf{Y}_0; t) \rangle = \frac{N_\cap \exp[-T'_W/\tau'_D - 2(T'_L - T'_W)/\tau'_D]}{N_L + N_R - N_\cap}, \quad (7)$$

where $T'_{L,W}$ are shorthand for $T'_{L,W}(r_{0\perp}, p_{0\perp})$. Note that the τ'_D

in the denominator of $\langle P'(\mathbf{Y}, \mathbf{Y}_0; t) \rangle$ was cancelled when we integrated over all times longer than $2T'_L$. For small $(p_{0\perp} + m\Lambda r_{0\perp})$ we find

$$T'_L(r_{0\perp}, p_{0\perp}) \simeq \Lambda^{-1} \ln \left[\frac{m\Lambda L}{|p_{0\perp} + m\Lambda r_{0\perp}|} \right] \quad (8)$$

and $T'_W(r_{0\perp}, p_{0\perp})$ is given by the same formula with L replaced by W . Evaluating the integrals over the relative coordinates $(r_{0\perp}, p_{0\perp})$ as in Ref. [1], we finally obtain

$$\langle \delta g \rangle_{\text{LR:1}} = N_{\cap} [2(N_L + N_R - N_{\cap})]^{-1} \exp[-\tau'_E/\tau'_D]. \quad (9)$$

The failed coherent forward-scattering contributions labeled 2i and 2ii in Fig. 2 come from the window of width $W_L - W_{\cap}$ in the L lead. This causes an enhanced probability of hitting the mirror image of that part of lead L. However, the lead R is not there, so this constructive interference peak gets reflected back into the dot and has a probability of $N_R/(N_L + N_R)$ of going to lead R and a probability of $N_L/(N_L + N_R)$ of going back to lead L [22]. The former is a contribution to transmission (and hence to the conductance) while the latter is a contribution to reflection. Thus, we have

$$\langle \delta g \rangle_{\text{LR:2i}} = \frac{(N_L - N_{\cap}) N_R \exp[-\tau'_E/\tau'_D]}{2(N_L + N_R - N_{\cap})(N_L + N_R)}, \quad (10)$$

$$\langle \delta R \rangle_{\text{LR:2ii}} = \frac{(N_L - N_{\cap}) N_L \exp[-\tau'_E/\tau'_D]}{2(N_L + N_R - N_{\cap})(N_L + N_R)}. \quad (11)$$

By inspection of Fig. 2 it follows that $\langle \delta g \rangle_{\text{LR:2iii}}$ is given by Eq. (10) with N_R and N_L interchanged, while $\langle \delta R \rangle_{\text{LR:2iv}} = \langle \delta R \rangle_{\text{LR:2ii}}$.

B. Uniform contributions to transmission and reflection

To evaluate the uniform contributions to transmission and reflection, labeled 3i and 3ii in Fig. 2, we divide the pairs of paths in this contribution into three regions. The first part is when γ and γ' are the same and are far from the encounter (a time $T_W/2$ or more away from the encounter). Here the probability for the paths to escape is $1/\tau_D$ per unit time. The second region is where γ' and γ are the mirror image of each other and far from the encounter (a time $T_W/2$ or more from the encounter). Here the probability of one or both paths to escape is $1/\tau'_D$ per unit time. Finally, the third region is close to the encounter (less than a time $T_W/2$ away from the encounter). Here the probability for the paths to escape the first time they pass through this region surrounding the encounter is $\exp[-T_W/\tau_D]$. However, the conditional probability to escape the second time the paths pass through this region (given that they both survived the first time) is $\exp[-T_W(1/\tau'_D - 1/\tau_D)]$. It follows that the contribution 3i is given by $\langle \delta g \rangle_{\text{LR:3i}} = (\pi\hbar)^{-1} \int_L d\mathbf{Y}_0 \int d\epsilon \text{Re} [e^{i(S_{\gamma} - S_{\gamma'})/\hbar}] \langle F(\mathbf{Y}_0, \epsilon) \rangle$, where the action difference $(S_{\gamma} - S_{\gamma'})$ is the same as for weak localization in Refs. [16,23] and

$$\begin{aligned} F(\mathbf{Y}_0, \epsilon) &= 2v_F^2 \sin \epsilon \int_{T_L+T_W}^{\infty} dt \int_{T_L+T_W/2}^{t-T_W/2} dt_2 \int_{T_W/2}^{t_2-T_L} dt_1 p_F \\ &\times \cos \theta_0 \int_R d\mathbf{Y} \int_C d\mathbf{R}_1 \tilde{P}(\mathbf{Y}, \mathbf{R}_2; t - t_2) \tilde{P}' \\ &\times (\mathbf{R}_2, \mathbf{R}_1; t_2 - t_1) \tilde{P}(\mathbf{R}_1, \mathbf{Y}_0; t_1). \end{aligned} \quad (12)$$

Since the paths are paired with their mirror image between time t_1 and time t_2 , the survival rate is τ'_D during this time, but it is τ_D at all other times. Evaluating this integral with these survival times gives

$$\begin{aligned} \langle F(\mathbf{Y}_0, \epsilon) \rangle &= \frac{2v_F^2 \tau_D \tau'_D}{2\pi A} \frac{N_R}{N_L + N_R} p_F \cos \theta_0 \sin \epsilon \\ &\times \exp[-T_L(\epsilon)/\tau'_D]. \end{aligned} \quad (13)$$

This has two differences from the result for symmetric leads in Ref. [1]. The exponent contains τ'_D not τ_D and the prefactor contains $\tau_D \tau'_D$ not τ_D^2 . When integrating over ϵ , we obtain a factor of $[\Lambda \tau'_D]^{-1} \exp[-\tau'_E/\tau'_D]$ in place of $[\Lambda \tau_D]^{-1} \exp[-\tau'_E/\tau_D]$. Thus, the τ'_D in the prefactor is cancelled [24]. Evaluating the integrals, we get

$$\langle \delta g \rangle_{\text{LR:3i}} = -N_L N_R [N_L + N_R]^{-2} \exp[-\tau'_E/\tau'_D], \quad (14)$$

$$\langle \delta R \rangle_{\text{LR:3ii}} = -N_L^2 [N_L + N_R]^{-2} \exp[-\tau'_E/\tau'_D]. \quad (15)$$

These results are of the same form as the weak localization correction except that the exponent contains τ'_D in place of τ_D . In particular, we recover the familiar factor of $-N_L N_R / (N_L + N_R)^2$ even though the joint survival time is reduced when the paths are mirror images of each other.

One can next include other suppression effects such as asymmetry in the dot and dephasing, which we discussed for dots with symmetric leads in Ref. [1]. The only difference caused by asymmetric leads is that now the parts of contributions affected by asymmetries and dephasing (parts where paths are paired with their mirror image) decay with a rate τ'_D instead of τ_D . Thus, we find that all the contributions listed in Fig. 2 are then multiplied by a factor

$$Z'_{\text{LR}}(\gamma_{\text{asym}}, \gamma_{\phi}) = \frac{\exp[-\gamma_{\phi} \tilde{\tau} - \gamma_{\text{asym}} \tilde{\tau}_{\text{asym}}]}{1 + (\gamma_{\text{asym}} + \gamma_{\phi}) \tau'_D}, \quad (16)$$

where the expression for the decay rates γ_{asym} and γ_{ϕ} and time scales $\tilde{\tau}$ and $\tilde{\tau}_{\text{asym}}$ are the same as for a dot with symmetric leads [1].

C. Conductance of a left-right symmetric quantum dot with asymmetric leads

As required by particle number conservation, the seven contributions in Fig. 2 sum to zero. In order to obtain the conductance, we sum the four contributions to transmission from the left lead to the right lead (contributions 1, 2i, 2iii, and 3i) and add them to the Drude conductance and the weak localization correction. This gives the conductance of a chaotic left-right symmetric quantum dot with many modes on each lead ($N_L, N_R, N_{\cap} \gg 1$),

$$\langle g \rangle_{\text{LR}} = \frac{N_L N_R}{N_L + N_R} + \frac{N_L N_R}{(N_L + N_R)^2} \left[\frac{N_{\cap} e^{-\tau_E^c / \tau_D'}}{N_L + N_R - N_{\cap}} Z'_{\text{LR}}(\gamma_{\text{asym}}, \gamma_{\phi}) - e^{-\tau_E^c / \tau_D} Z(B, \gamma_{\phi}) \right] + \mathcal{O}[N_{\text{L,R}}^{-1}], \quad (17)$$

where $Z'_{\text{LR}}(\gamma_{\text{asym}}, \gamma_{\phi})$ is given by Eq. (16). The second term in the square brackets is the usual weak localization correction, which is suppressed by magnetic fields and dephasing according to the function $Z(B, \gamma_{\phi}) = \exp[-\gamma_{\phi} \tilde{\tau} [1 + (B/B_c)^2 + \gamma_{\phi} \tau_D]]^{-1}$. For symmetric leads we have $N_{\cap} = N_L = N_R$ (and hence $\tau_D' = \tau_D$) and this result immediately reduces to the one in Ref. [1].

It is worth considering two special cases. The first case is when the leads are of equal width but not centered at the mirror image of each other such that $N_{\cap} < N_L = N_R \equiv N$. Taking $\Delta w = 1 - w_{\cap} / W = 1 - N_{\cap} / N$ as the relative distance (in units of the lead width $W = W_L = W_R$) by which lead L is displaced with respect to the mirror image of lead R, and assuming there is no dephasing, magnetic field, or internal asymmetry, we find

$$\langle g \rangle_{\text{LR}} = \frac{N}{2} + \frac{1}{4} \left[\frac{1 - \Delta w}{1 + \Delta w} e^{-\tau_E^c / \tau_D'} - e^{-\tau_E^c / \tau_D} \right] + \mathcal{O}[N^{-1}]. \quad (18)$$

The second special case is when the lead R is narrower but situated entirely within the mirror image of lead L; we then have $N_{\cap} = N_R < N_L$. Assuming again that there is no dephasing, magnetic field, or internal asymmetry,

$$\langle g \rangle_{\text{LR}} = \frac{N_L N_R}{N_L + N_R} + \frac{N_L N_R}{(N_L + N_R)^2} \left[\frac{N_R}{N_L} e^{-\tau_E^c / \tau_D'} - e^{-\tau_E^c / \tau_D} \right] + \mathcal{O}[N_{\text{L,R}}^{-1}]. \quad (19)$$

As one could scan the narrow lead R across the mirror image of the wide lead L, this scenario can be thought of as a probe of the shape of the coherent forward-scattering peak. The fact that our result Eq. (19) is independent of the position of lead R tells us that the forward-scattering peak is uniformly distributed over the region defined by the mirror image of lead L.

IV. INVERSION-SYMMETRIC QUANTUM DOT WITH ASYMMETRIC LEADS

For systems with inversion symmetry the calculation follows much as for a left-right symmetry. The one significant difference is the magnetic-field dependence of the contributions, which was treated in Ref. [1]. The displacement of the leads simply requires us to replace τ_D with τ_D' in the suppression of contributions by magnetic fields, asymmetries in the dot, and dephasing. The suppression factor, therefore, takes the form

$$Z'_{\text{inv}}(B, \gamma_{\text{asym}}, \gamma_{\phi}) = \frac{\exp[-\gamma_{\text{asym}} \tilde{\tau}_{\text{asym}} - \gamma_{\phi} \tilde{\tau}]}{1 + (B/B_c')^2 + (\gamma_{\text{asym}} + \gamma_{\phi}) \tau_D'}, \quad (20)$$

where $B_c' = aB_0 \sqrt{\tau_0 / 2 \tau_D'}$ is given by Eq. (1) with τ_D replaced by τ_D' . As a result, an inversion-symmetric quantum dot with

many modes on each lead ($N_L, N_R, N_{\cap} \gg 1$) has a total average conductance of

$$\langle g \rangle_{\text{inv}} = \frac{N_L N_R}{N_L + N_R} + \frac{N_L N_R}{(N_L + N_R)^2} \times \left[\frac{N_{\cap} e^{-\tau_E^c / \tau_D'}}{N_L + N_R - N_{\cap}} Z'_{\text{inv}}(B, \gamma_{\text{asym}}, \gamma_{\phi}) - e^{-\tau_E^c / \tau_D} Z(B, \gamma_{\phi}) \right] + \mathcal{O}[N_{\text{L,R}}^{-1}]. \quad (21)$$

With the exception of the magnetic-field dependence of the second term, this formula is the same as Eq. (17) for a left-right symmetric dot. Thus, the two special cases discussed below Eq. (17) are directly applicable here.

V. UP-DOWN SYMMETRIC QUANTUM DOT WITH ASYMMETRIC LEADS

For up-down symmetric systems, there are a number of important differences with the case of left-right symmetry discussed in Sec. III. First, a pair of paths related by the mirror symmetry decays jointly at a rate

$$\tau_D^{(\text{UD})} = \tau_D \frac{N_L + N_R}{2N_L + 2N_R - N_{\cap L} - N_{\cap R}}, \quad (22)$$

where $N_{\cap L}$ is the number of modes in the intersection of lead L with its own mirror image and $N_{\cap R}$ is the number of modes in the intersection of lead R with its own mirror image. Second, the successful and failed forward-scattering contributions for left-right symmetry are converted into successful and failed backscattering contributions for up-down symmetry. In particular, successful backscattering makes no contribute to the conductance. The other contributions to transmission are not very different from those for left-right symmetry except that one must distinguish $N_{\cap L}$ from $N_{\cap R}$ and one must replace τ_D' by $\tau_D^{(\text{UD})}$. Summing up the contributions to conductance induced by the spatial symmetry, we find

$$\langle \delta g \rangle_{\text{UD}} = - \frac{(N_{\cap L} N_R^2 + N_{\cap R} N_L^2) \exp[-\tau_E^c / \tau_D^{(\text{UD})}]}{(2N_L + 2N_R - N_{\cap L} - N_{\cap R})(N_L + N_R)^2} \times Z'_{\text{UD}}(\gamma_{\text{asym}}, \gamma_{\phi}), \quad (23)$$

where $Z'_{\text{UD}}(\gamma_{\text{asym}}, \gamma_{\phi})$ has the same form as $Z'_{\text{LR}}(\gamma_{\text{asym}}, \gamma_{\phi})$ given in Eq. (16), but with τ_D' replaced by $\tau_D^{(\text{UD})}$. Like for left-right mirror symmetry (but unlike for inversion symmetry) this contribution is unaffected by a magnetic field.

The average conductance of an up-down mirror-symmetric dot with many modes on each lead is therefore

$$\langle g \rangle_{\text{UD}} = \frac{N_L N_R}{N_L + N_R} - \frac{N_L N_R}{(N_L + N_R)^2} \left[\left(\frac{N_{\cap L} N_R}{N_L} + \frac{N_{\cap R} N_L}{N_R} \right) \times \frac{\exp[-\tau_E^c / \tau_D^{(\text{UD})}]}{2N_L + 2N_R - N_{\cap L} - N_{\cap R}} Z'_{\text{UD}}(\gamma_{\text{asym}}, \gamma_{\phi}) + \exp[-\tau_E^c / \tau_D] Z(B, \gamma_{\phi}) \right] + \mathcal{O}[N_{\text{L,R}}^{-1}]. \quad (24)$$

It is worth noting that the spatial symmetry induces a reduction of conductance whenever one lead is close to sym-

metric, even if the other lead is completely asymmetric (i.e., when $N_{\cap L}=0$ but $N_{\cap R} \neq 0$, or vice versa). For example, when both leads have the same width ($N_L=N_R=N$) and the right lead is perfectly on the symmetry axis ($N_{\cap R}=N_R$), but the left lead is a long way from the symmetry axis ($N_{\cap L}=0$), Eq. (24) reduces to

$$\langle g \rangle_{\text{UD}} = \frac{N}{2} - \frac{1}{4} \left\{ \frac{1}{3} \exp[-\tau_E^c/\tau_D^{\text{UD}}] + \exp[-\tau_E^c/\tau_D] \right\} + \mathcal{O}[N^{-1}] \quad (25)$$

assuming no dephasing, magnetic field, and no asymmetry in the dot. If the Ehrenfest time is much shorter than τ_D and τ_D^{UD} , the average conductance of the system with one displaced lead is, therefore, simply $\langle g \rangle_{\text{UD}}=N/2-1/3$.

Remarkably, the conductance from the L lead to the R lead is, therefore, affected by the symmetry of the dot even when the L lead is completely asymmetric. This result is perhaps less counterintuitive when one considers reflection (rather than transmission). If one lead is on the symmetry axis, then reflection back to that lead will be enhanced even if the other lead is a long way from the symmetry axis. Since we have particle conservation, there must be an associated reduction in transmission from one lead to the other (compared to transmission in a completely asymmetric situation).

VI. FOURFOLD SYMMETRIC QUANTUM DOT WITH ASYMMETRIC LEADS

A quantum dot with fourfold symmetry simultaneously possesses all three of the spatial symmetries that we discuss in this paper. The interference corrections to the conductance of such a system are simply the sum of the corrections due to each of these three symmetries (i.e., the presence of the extra symmetries has no effect on the contributions which do not respect those symmetries),

$$\langle \delta g \rangle_{4F} = \langle \delta g \rangle_{\text{LR}} + \langle \delta g \rangle_{\text{inv}} + \langle \delta g \rangle_{\text{UD}}, \quad (26)$$

where $\langle \delta g \rangle_{\kappa}$ is the contribution to the average conductance induced by spatial symmetry $\kappa \in \text{LR, inv, UD}$. The explicit form of this result is easily extracted from the expressions in the previous sections. Instead of writing it out in full, we consider the special case where the two leads have the same width, $N_L=N_R=N$, the Ehrenfest time is negligible and there is no dephasing, magnetic field, or asymmetry in the dot. The average conductance then takes the form

$$\langle g \rangle_{4F} = \frac{N}{2} + \frac{1}{4} \left[\frac{N_{\cap \text{LR}}}{2N - N_{\cap \text{LR}}} + \frac{N_{\cap \text{inv}}}{2N - N_{\cap \text{inv}}} - \frac{N_{\cap \text{UD:L}} + N_{\cap \text{UD:R}}}{4N - N_{\cap \text{UD:L}} - N_{\cap \text{UD:R}}} - 1 \right], \quad (27)$$

where $N_{\cap \text{LR}}$ is the intersection between leads L and R under the left-right symmetry, $N_{\cap \text{inv}}$ is the intersection between leads L and R under the inversion symmetry, and $N_{\cap \text{UD:L}}$ ($N_{\cap \text{UD:R}}$) is the intersection of lead L (R) with itself under the up-down symmetry. The final term in the square bracket is the usual weak localization contribution.

Since the presence of two of the above-mentioned symmetries always implies the presence of the third, it is not

possible to move the leads such that only one of the N_{\cap} parameters changes. Without affecting the integrity of the leads there are only two possible modifications for which only two of the parameters change; starting with perfectly symmetric leads one can (a) move both leads upward by the same amount so that $N_{\cap \text{LR}}$ is unchanged or (b) move both leads by the same amount in opposite directions (one up and one down) so that $N_{\cap \text{inv}}$ is unchanged. In principle, it is also possible to break up a single lead (say L) in the middle and move the two parts into opposite directions (both parts would still be contacted by the same source or drain electrode); this preserves $N_{\cap L}$ and $N_{\cap R}$ but affects the other parameters. However, the latter deformation is difficult to realize in practice.

VII. UNIVERSAL CONDUCTANCE FLUCTUATIONS WITH DISPLACED LEADS

Now we turn to the magnitude of UCFs in symmetric dots with asymmetric leads. Their calculation is generally far more complicated than the calculation of the average conductance. This is illustrated by the fact that there is as yet no semiclassical theory of UCFs for leads with tunnel barriers, a problem which has many similarities to the problem we need to solve here. Thus, we restrict ourselves to the simplest case of quantum dots with negligible Ehrenfest time and negligible dephasing, and only consider magnetic fields which are either negligibly small ($B \ll B_c$), or sufficiently strong to break time-reversal symmetry in the asymmetric system ($B \gg B_c$).

The magnitude of the UCFs (with conductances measured in units of e^2/h) is given by $\text{var}[g]=\text{var}[T]$, where $T=\text{tr}[t^\dagger t]$ and t is the block of the scattering matrix $S=\begin{pmatrix} r & t' \\ t & r' \end{pmatrix}$ associated with transmission from lead L to lead R. For practical calculations it is beneficial to exploit the unitarity of the scattering matrix (i.e., current conservation), which results in the relations $T=N_L-R=N_R-R'$ with $R=\text{tr}[r^\dagger r]$ and $R'=\text{tr}[r'^\dagger r']$, where r is the block of the scattering matrix associated with reflection back to lead L and r' describes reflection back to lead R. As a result we can write the magnitude of the UCFs in any of the following ways:

$$\text{var}[g] = \text{var}[R] = \text{var}[R'] = \text{covar}[R, R']. \quad (28)$$

As for conventional UCFs without spatial symmetries [19,20], the semiclassical calculation of $\text{covar}[R, R']$ is most straightforward; thus, we base our calculations on this quantity. For the expert reader, the Appendix contains an outline of the calculation of $\text{var}[R]$ and $\text{var}[R']$ showing that they equal $\text{covar}[R, R']$.

All symmetry-induced contributions to $\text{covar}[R, R']$ for an up-down symmetric dot are listed in Fig. 4. For a left-right or inversion-symmetric dot there are *additional* contributions, which are listed in Fig. 5. In all cases, when paths 2 and 2' are not paired with each other, they are paired with the *images* of paths 1' and 1 under the appropriate symmetry operation. To keep the sketches in Figs. 4 and 5 as clear as possible, we only show these images of paths 1 and 1' (rather than paths 1 and 1' themselves). Then the resulting contributions look very much like the usual contributions to

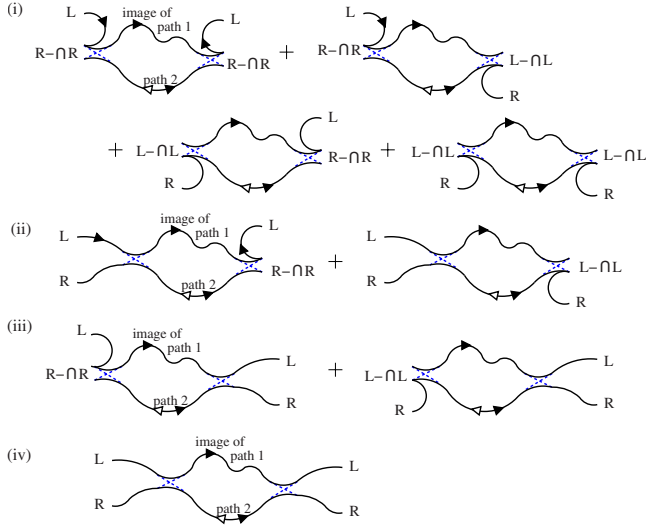


FIG. 4. (Color online) A sketch of semiclassical contributions to UCFs (more specifically, contributions to $\text{covar}[R, R']$) for an up-down symmetric dot with asymmetric leads. There are analogous contributions to UCFs for left-right or inversion-symmetric dots (see explanation in the text). In each contribution, paths 1 and 1' go from L lead to L lead, while paths 2 and 2' go from R lead to R lead. In the sketches, solid lines indicate paths 2 and the image (mirror image or image under the inversion symmetry) of paths 1. Path 2' and the image of path 1' are indicated by the dashed lines (only shown at the encounters). Thus when paths 2 and 2' are not paired with each other they are paired with the image of 1' and 1, respectively (indicated by solid arrowheads). If the system has a time-reversal symmetry then paths 2 and 2' can also be paired with the time reversals of the image of 1' and 1, respectively (indicated by the open arrowheads).

UCFs in a system without a spatial symmetry [19,20].

In analogy to the situation in asymmetric systems, one would also expect contributions in which paths wind around periodic orbits (see Figs. 1b and 1c in Ref. [20]). For example, a symmetric quantum dot will have contributions in which path 1' is the same as path 1 except that it winds around a periodic orbit p when path 1 does not (thus path 1 must come very close to the periodic orbit in phase space), while path 2 is the same as path 2' except that it winds around the *image* of the orbit p . These contributions are proportional to those analyzed for UCFs in asymmetric dots, the only modification being that the joint survival probability of a periodic orbit and its image is again changed to $\exp[-t/\tau_D']$. Drawing on the results of Refs. [19,20], it follows that the contributions involving windings around periodic orbits will be negligibly small when the Ehrenfest time is small. (This observation makes the calculation of the UCFs in the present problem significantly simpler than for the case with tunnel barriers, where one cannot rule out contributions from periodic orbits which touch the barriers on the leads.)

A. Effect of time-reversal symmetry

Inspecting the sketches in Figs. 4 and 5 we see that all contributions are doubled when the magnetic field is negli-

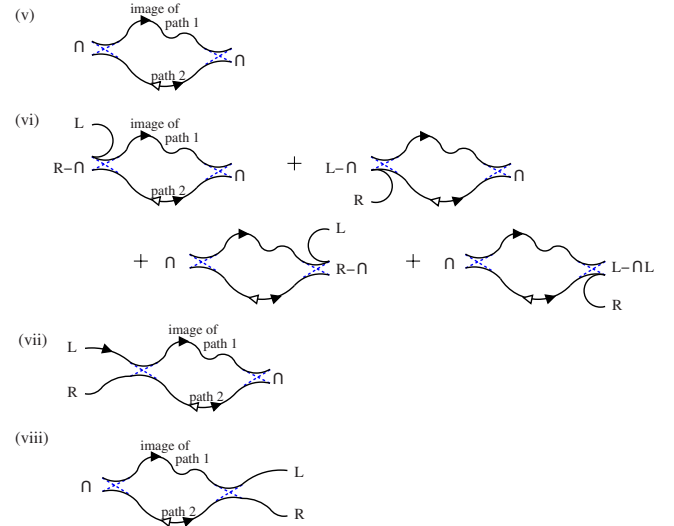


FIG. 5. (Color online) A sketch of additional semiclassical contributions to UCFs (more specifically, contributions to $\text{covar}[R, R']$) for left-right or inversion-symmetric dots with asymmetric leads. The contributions listed here must be *added* to those listed in Fig. 4 (once one sets $\cap L = \cap R = \cap$) to get the full set of contributions for left-right or inversion-symmetric dots. The manner in which the contributions are sketched is explained in the caption of Fig. 4.

gible, because path 2 can either follow the image of path 1' or the time reverse of path 1'. Thus, we can multiply all terms by $2/\beta$, where $\beta=1$ for a system with negligible magnetic field, $B \ll B_c$ and $\beta=2$ for a system with a finite magnetic field, $B \gg B_c$. In the latter case the presence of mirror-reflection symmetries allows one to define a generalized time-reversal symmetry; however, this is already accounted for in the construction of all diagrams (see Appendix A of Ref. [1]).

B. UCFs in an up-down symmetric dot

The general rules for constructing all contributions to the UCFs are the following. Each segment where path 2 or 2' is paired with the image of path 1' or 1 gives a factor of $(2N_L + 2N_R - N_{\cap L} - N_{\cap R})^{-1}$, which arises from the survival time τ_D^{UD} given in Eq. (22). Each segment where paths 2 and 2' are paired (or paths 1 and 1' are paired) gives a factor of $(N_L + N_R)^{-1}$, which comes from the conventional survival time τ_D . Each segment that touches a lead gives a factor equal to the number of lead modes that the path could couple to; i.e., a lead labeled “ $R - \cap R$ ” gives a factor of $(N_R - N_{\cap R})$, while a lead labeled “ R ” simply gives a factor of N_R . An encounter which touches a lead gives the same factor as a simple path segment that touches a lead, so again if it is labeled “ $R - \cap R$ ” then it gives a factor of $(N_R - N_{\cap R})$. (This rule is proven by applying the same analysis as was used for the successful and failed forward-scattering processes in Sec. III A.) Finally, encounters deep in the dot (i.e., those which do not touch the leads) give a factor of $-(2N_L + 2N_R - N_{\cap L} - N_{\cap R})$ (this rule can be proven by applying the same analysis as was used for the uniform contributions to transmission in Sec. III B). With this set of rules we can easily see that contribution (i) in Fig. 4 gives

$$C_i = \frac{2N_L^2(N_R - N_{\cap R})^2 + 2N_L(N_L - N_{\cap L})N_R(N_R - N_{\cap R}) + (N_L - N_{\cap L})^2N_R^2}{\beta(2N_L + 2N_R - N_{\cap L} - N_{\cap R})^2(N_L + N_R)^2}. \quad (29)$$

Next we see that $C_{iii}=C_{ii}$ and that they are negative because only one of the encounters is deep in the dot (the other is near a lead) resulting in

$$C_{ii} + C_{iii} = -2 \frac{2N_L^2N_R(N_R - N_{\cap R}) + N_L(N_L - N_{\cap L})N_R^2}{\beta(2N_L + 2N_R - N_{\cap L} - N_{\cap R})(N_L + N_R)^3}. \quad (30)$$

Finally C_{iv} gives a positive contribution because it has two encounters deep in the dot and is given by

$$C_{iv} = \frac{2}{\beta} \frac{N_L^2N_R^2}{(N_L + N_R)^4}. \quad (31)$$

The total magnitude of the UCFs is given by the UCFs of an asymmetric dot, $\text{var}[g]_{\text{asym}}$, plus the sum of the terms above, i.e., $\text{var}[g] = \text{var}[g]_{\text{asym}} + C_i + C_{ii} + C_{iii} + C_{iv}$. In the limit of perfectly symmetric leads ($N_{\cap L}=N_L$ and $N_{\cap R}=N_R$), only C_{iv} survives and the UCFs have double the magnitude as those for an asymmetric dot. In the limit of completely asymmetric leads ($N_{\cap L}=N_{\cap R}=0$), one has $C_i + C_{ii} + C_{iii} + C_{iv} = 0$ and the UCFs have the same magnitude as those for an asymmetric dot.

To express $\text{var}[g]$ for arbitrary N_L , N_R , $N_{\cap L}$, and $N_{\cap R}$, we find it beneficial to introduce the quantities $n_\kappa = N_\kappa / (N_L + N_R)$ and $w_\kappa = 1 - N_{\cap \kappa} / N_\kappa$, where $\kappa = L, R$. Making use of the fact that $n_L + n_R = 1$, we find

$$\text{var}[g] = \text{var}[g]_{\text{asym}} + \frac{2}{\beta} n_L^2 n_R^2 \left(\frac{1 - (1 - n_L)w_L - (1 - n_R)w_R}{1 + n_L w_L + n_R w_R} \right)^2, \quad (32)$$

where in this notation $\text{var}[g]_{\text{asym}} = (2/\beta)n_L^2 n_R^2$. In the special case where $N_L = N_R$, displacing the leads suppresses the symmetry-induced contribution to UCFs by a factor $[(2 - w_L - w_R)/(2 + w_L + w_R)]^2$.

In terms of the original quantities N_L , N_R , $N_{\cap L}$, and $N_{\cap R}$, Eq. (32) takes the form

$$\begin{aligned} \text{var}[g] &= \text{var}[g]_{\text{asym}} \\ &+ \frac{2}{\beta} \frac{N_L^2 N_R^2}{(N_L + N_R)^4} \left(\frac{N_R N_{\cap L} / N_L + N_L N_{\cap R} / N_R}{2N_L + 2N_R - N_{\cap L} - N_{\cap R}} \right)^2, \end{aligned} \quad (33)$$

where $\text{var}[g]_{\text{asym}} = (2/\beta)N_L^2 N_R^2 (N_L + N_R)^{-4}$. Comparison with Eq. (24) shows that lead displacement suppresses the symmetry-induced contributions to UCFs by a factor that is the square of the suppression of the symmetry-induced contributions to the average conductance.

C. UCFs in a left-right or inversion-symmetric quantum dot

For a systems with a left-right or an inversion symmetry, we once again find the magnitude of the UCFs by evaluating

$\text{covar}[R, R']$. For these symmetries, we must consider the contributions in Fig. 5 in addition to those in Fig. 4. The origin of the extra contributions in Fig. 5 is most clearly understood by considering the case of perfectly symmetric leads. Then the left-right and inversion symmetries map lead L onto lead R, which means that if path 2 is paired with path 1' then path 2 will hit lead R when path 1' hits lead L (meaning the image of path 1' hits lead R). One can thereby immediately see that the contribution C_v in Fig. 5 contributes to $\text{covar}[R_L, R_R]$ (this was not the case for up-down symmetry, since there path 1' hits the same lead as the image of path 1'). For asymmetric leads a similar situation occurs. If path 1' hits the intersection region of width W_\cap on lead L then its image hits lead R; thus, path 2 will also hit lead R if it is paired with 1' over this segment.

The rules to evaluate each contribution are the same as for up-down symmetry, with now necessarily $N_{\cap L} = N_{\cap R} = N_\cap$. Using these rules, we find that

$$C_v + C_{vi} = \frac{2}{\beta} \frac{4N_\cap N_L N_R - N_\cap^2 (N_L + N_R)}{(2N_L + N_R - 2N_\cap)^2 (N_L + N_R)}, \quad (34)$$

$$C_{vii} + C_{viii} = -\frac{2}{\beta} \frac{2N_\cap N_L N_R}{(2N_L + N_R - 2N_\cap)(N_L + N_R)^2}. \quad (35)$$

Summing these contributions and writing the result with the same denominator as Eq. (33) gives

$$C_v + C_{vi} + C_{vii} + C_{viii} = -\frac{2}{\beta} \frac{N_\cap^2 (N_L^2 - N_R^2)^2}{(2N_L + N_R - 2N_\cap)^2 (N_L + N_R)^4}. \quad (36)$$

Adding this set of contribution to those already calculated in the previous section, we find that the UCFs of a left-right or inversion-symmetric dot with asymmetric leads is given by

$$\text{var}[g] = \text{var}[g]_{\text{asym}} + \frac{2}{\beta} \frac{N_L^2 N_R^2}{(N_L + N_R)^4} \left(\frac{N_\cap}{N_L + N_R - N_\cap} \right)^2. \quad (37)$$

By comparing this with Eq. (17), we find that the suppression of symmetry-induced contributions to UCFs is the square of suppression of the symmetry-induced contributions to the average conductance (just as we already found for an up-down symmetric system).

D. UCFs in a fourfold symmetric

For completeness, we now briefly discuss UCFs in a fourfold symmetric dot with asymmetric leads. A fourfold dot has all three of the symmetries discussed above. Thus, the UCFs in a fourfold symmetric system are given by the sum of all possible symmetry-induced contributions (just as with

symmetric leads [1]). Given the results in the preceding sections, the general formula is easily determined. Here we give the result for the special case $N_L = N_R = N$,

$$\text{var}[g] = \frac{1}{8\beta} \left[\left(\frac{N_{\cap LR}}{2N - N_{\cap LR}} \right)^2 + \left(\frac{N_{\cap \text{inv}}}{2N - N_{\cap \text{inv}}} \right)^2 + \left(\frac{N_{\cap \text{UD:L}} + N_{\cap \text{UD:R}}}{4N - N_{\cap \text{UD:L}} - N_{\cap \text{UD:R}}} \right)^2 + 1 \right], \quad (38)$$

where $N_{\cap LR}$ is the intersection between leads L and R under the left-right symmetry, $N_{\cap \text{inv}}$ is the intersection between leads L and R under the inversion symmetry, and $N_{\cap \text{UD:L}}$ ($N_{\cap \text{UD:R}}$) is the intersection of lead L (R) with itself under the up-down symmetry. The final term in the square bracket represents the usual UCFs for an asymmetric dot. Note that the suppression of each symmetry-induced term goes like the square of the equivalent term in the average conductance, Eq. (27).

VIII. COMPARISON TO RANDOM-MATRIX THEORY

In this section we compare the semiclassical predictions derived in the previous sections to numerical results obtained from a phenomenological random-matrix model. This model generalizes the construction discussed in Sec. 9 of part I (Ref. [1]).

The general framework is the same as in part I: the conductance is obtained from the Landauer formula $g = \text{tr}[t^\dagger t]$, where t is the transmission block of a scattering matrix $S = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix}$ given by

$$S = P^T(1 - FQ)^{-1}FP. \quad (39)$$

Here, F is an internal unitary evolution operator of dimension M while P is an $M \times 2N$ dimensional matrix specified below and $Q = 1 - PP^T$.

In part I we assumed that the leads respect the geometrical symmetries, which allows to fully desymmetrize the system. One can then introduce a fixed form of the matrix P and attribute the effects of symmetries solely to the internal dynamics (the resulting RMT ensembles for F are given in Table II of Ref. [1]). It is clear that this full desymmetrization fails when leads are to be displaced. For up-down symmetry, for instances, desymmetrization identifies two effectively separate systems (consisting of modes of even and odd parity) which do not couple to each other. Shifting lead modes in this representation has no effect since RMT is invariant under the permutation of matrix indices. A real displacement of leads, however, mixes the states of even and odd parity. The reason for this discrepancy is that leads are defined locally in real space, while parity is a global symmetry which connects remote parts of the system.

It is therefore necessary to define both the internal evolution operator F as well as the coupling to the leads P in a way which resembles modes in a real-space basis. In principle, this can be done, e.g., based on the sinusoidal transverse mode profiles of a strip resonator. We adopt a similar but more efficient procedure whose principle idea is shown in Fig. 6. The illustration shows an abstract scatterer with M

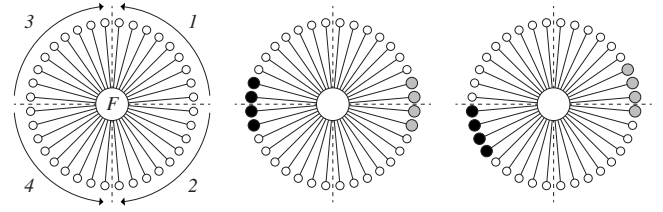


FIG. 6. Left panel: model of a scatterer (central circle) with internal evolution operator F coupled to ports to which modes of the leads can be attached. The labels identify four segments in which the ports are numerated in the direction of the arrow (port 1 to $M/4$ in segment 1, port $M/4+1$ to $M/2$ in segment 2, port $M/2+1$ to $3M/4$ in segment 3, and port $3M/4+1$ to M in segment 4). The dashed lines indicate the possible lines of reflection symmetry. Middle and right panels: filled circles indicate ports coupled to the left lead; shaded circles indicate ports coupled to the right lead. Shown are a fully symmetry-respecting arrangement and an arrangement in which both leads are displaced, respectively.

ports which serve as possible contacts to the system. For each lead we select N ports (with index i_n for lead L and j_n for lead R); the remaining ports are closed off. The internal evolution operator F describes the transport from port to port. The scattering matrix is then given by Eq. (39) where $P_{mn} = \delta_{m,i_n} + \delta_{m,j_n-N}$.

A crucial point of the illustration in Fig. 6 is the numeration of ports, which are grouped into four segments that map in specific ways onto each other when symmetry operations are applied. (i) Left-right symmetry maps segment 1 onto segment 3 and segment 2 onto segment 4. (ii) Up-down symmetry maps segment 1 onto segment 2 and segment 3 onto segment 4. (iii) Inversion symmetry maps segment 1 onto segment 4 and segment 2 onto segment 3. (iv) Fourfold symmetry maps all segments onto each other.

In the basis of these ports, the explicit symmetries of the internal evolution operator F are specified in Table I. Since up-down and left-right symmetries are both manifestations of a reflection symmetry, they are now simply related by a interchanging segments 2 and 3 (as described by the matrix C defined in Table I); this is a consequence of the fact that we do not fully desymmetrize the up-down symmetry (the left-right symmetric case can never be fully desymmetrized because one has to keep track of the identity of the leads). A finite magnetic field breaks these symmetries but still allows one to define a generalized time-reversal symmetry. Similarly, for vanishing magnetic field, inversion symmetry is obtained from reflection symmetry by interchanging segments 3 and 4 (as described by the matrix D defined in the table caption). The slightly different systematics in the presence of a magnetic field arises because the orientation of the segments matters; consequently, for inversion symmetry, time-reversal symmetry is effectively broken but the geometric symmetry itself is still present in the dynamics (trajectories still occur in symmetry-related pairs).

A convenient choice of a fully symmetry-respecting arrangement of leads which applies to all internal symmetries is given by

TABLE I. Random-matrix ensembles for the internal evolution operator F in a basis which is suitable for displacing the leads (see Fig. 6). The different entries refer to various geometric symmetries in absence or presence of a magnetic field. We only consider the case $M \bmod 4=0$. Block composition of two identical matrix ensembles of dimension M is abbreviated as $X^2(M)=X(M) \otimes X(M)$. COE denotes the circular orthogonal ensemble while CUE denotes the circular unitary ensemble.

	$B=0$	$B \gg B_c$
No spatial sym.	COE(M)	CUE(M)
Left-right sym.	$A^\dagger \text{COE}^2(M/2) A$	$A^\dagger \text{COE}(M) A$
Inversion sym.	$DA^\dagger \text{COE}^2(M/2) AD$	$DA^\dagger \text{CUE}^2(M/2) AD$
Up-down sym.	$CA^\dagger \text{CUE}^2(M/2) AC$	$CA^\dagger \text{COE}(M) AC$
Fourfold sym.	$DA^\dagger [A^\dagger \text{CUE}^2(M/4) A]^2 AD$	$DA^\dagger [A^\dagger \text{COE}(M/2) A]^2 AD$

with $A = 2^{-1/2} \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}$,

$C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

and $D = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

$$P = \begin{pmatrix} 1_{\mathcal{N} \times \mathcal{N}} & 0_{\mathcal{N} \times \mathcal{N}} & 0_{\mathcal{N} \times \mathcal{N}} & 0_{\mathcal{N} \times \mathcal{N}} \\ 0_{\mathcal{M} \times \mathcal{N}} & 0_{\mathcal{M} \times \mathcal{N}} & 0_{\mathcal{M} \times \mathcal{N}} & 0_{\mathcal{M} \times \mathcal{N}} \\ 0_{\mathcal{N} \times \mathcal{N}} & 1_{\mathcal{N} \times \mathcal{N}} & 0_{\mathcal{N} \times \mathcal{N}} & 0_{\mathcal{N} \times \mathcal{N}} \\ 0_{\mathcal{M} \times \mathcal{N}} & 0_{\mathcal{M} \times \mathcal{N}} & 0_{\mathcal{M} \times \mathcal{N}} & 0_{\mathcal{M} \times \mathcal{N}} \\ 0_{\mathcal{N} \times \mathcal{N}} & 0_{\mathcal{N} \times \mathcal{N}} & 1_{\mathcal{N} \times \mathcal{N}} & 0_{\mathcal{N} \times \mathcal{N}} \\ 0_{\mathcal{M} \times \mathcal{N}} & 0_{\mathcal{M} \times \mathcal{N}} & 0_{\mathcal{M} \times \mathcal{N}} & 0_{\mathcal{M} \times \mathcal{N}} \\ 0_{\mathcal{N} \times \mathcal{N}} & 0_{\mathcal{N} \times \mathcal{N}} & 0_{\mathcal{N} \times \mathcal{N}} & 1_{\mathcal{N} \times \mathcal{N}} \\ 0_{\mathcal{M} \times \mathcal{N}} & 0_{\mathcal{M} \times \mathcal{N}} & 0_{\mathcal{M} \times \mathcal{N}} & 0_{\mathcal{M} \times \mathcal{N}} \end{pmatrix}, \quad (40)$$

where $\mathcal{N}=N/2$ and $\mathcal{M}=M/4-N/2$. The case of a fourfold symmetry in principle allows two symmetry-respecting arrangements (aligned along each of the two symmetry lines of reflection); these two arrangements are equivalent in RMT and again related by a reshuffling of the four segments. The form of P for generally placed leads is easily read off Fig. 6.

Figure 7 (for $B \ll B_c$) and Fig. 8 (for $B \gg B_c$) show how the weak localization correction and universal conductance fluctuations are affected when the leads are moved away from the symmetry-respecting positions. The degree of displacement is quantified by a variable $\lambda=1-W_\cap/W$ ($\lambda=0$ in the symmetric arrangement; $\lambda=1$ in the asymmetric arrangement). The data points are based on an ensemble average over 5000 RMT matrices with $M=1000$ and $N=50$, while the curves are the predictions of our semiclassical theory, which can be written as

$$\delta g(\lambda) = \delta g(1) + [\delta g(0) - \delta g(1)] \frac{1-\lambda}{1+\lambda}, \quad (41)$$

$$\text{var } g(\lambda) = \text{var } g(1) + [\text{var } g(0) - \text{var } g(1)] \left(\frac{1-\lambda}{1+\lambda} \right)^2. \quad (42)$$

Starting from a fourfold symmetry, leads can be displaced in a manner which still preserves left-right, inversion, or up-down symmetry. To preserve up-down symmetry alone, one

can imagine splitting one lead in two and moving the two parts in opposite directions (both parts would remain contacted to the same source or drain electrode). The remaining symmetry of the lead arrangement can then be broken by further displacement of the leads. In the figures, the subscript $_4$ is used to distinguish these situations (in which the underlying internal symmetry is fourfold) from the symmetry breaking in systems with only a single internal symmetry. E.g., the label “left/right $_4 \rightarrow$ asymmetric $_4$ ” refers to the displacement of leads out of a left-right symmetric position where the internal symmetry is fourfold, while the label “left/right \rightarrow asymmetric” refers to the displacement of leads out of a left-right symmetric position where the internal symmetry is itself only left-right symmetric. According to our theory, the weak localization correction should behave identically in both situations; this also applies to the UCFs. This statement is validated by the numerical data. Indeed, excellent agreement of the numerical data with the semiclassical predictions is observed in all cases.

As discussed earlier in this paper, in the up-down symmetric case it is interesting to displace only one lead while the other lead remains on the symmetry line (the symmetry-preserving positions in the up-down symmetric case are absolute, in contrast to the left-right symmetric case where these positions are relative to each other). The effect on the transport is shown in Fig. 9, along with the effect of the consecutive displacement of the second lead, and the simultaneous displacement of both leads. According to our theory, the effects of consecutive displacement of the leads are cumulative: the displacement of the first lead is described by Eqs. (41) and (42) with $\lambda \rightarrow \lambda/2$ (covering the range $[0,1/2]$), while the displacement of the second lead completes the transition according to the substitution $\lambda \rightarrow (1+\lambda)/2$ (covering the range $[1/2,1]$). The numerical results are in perfect agreement with this prediction.

We conclude with some additional remarks on the RMT model. For leads which respect the symmetries, the construction presented here is equivalent to the model presented in part I (which then is more efficient); this equivalence also

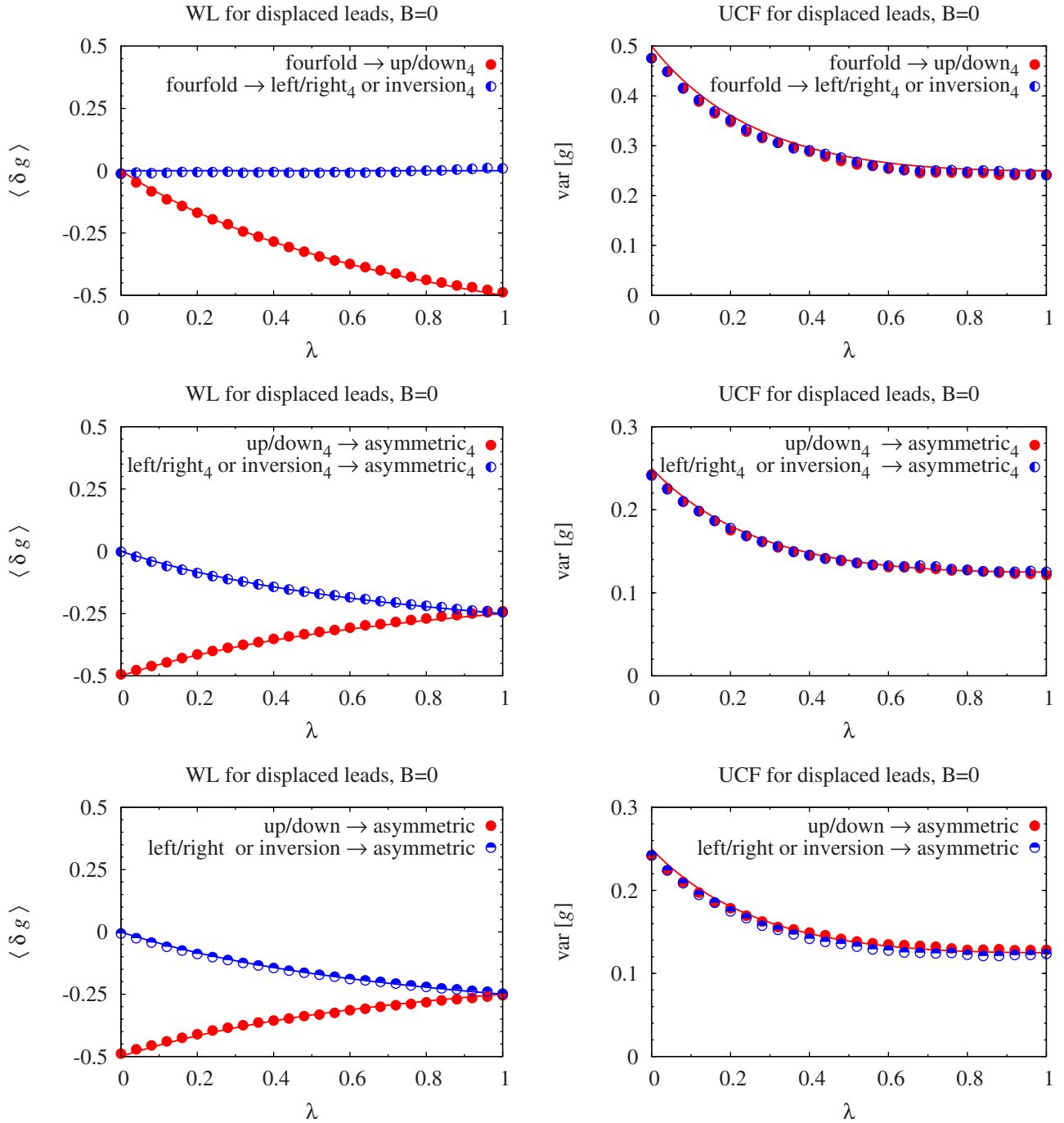


FIG. 7. (Color online) Weak localization correction (WL, left panels) and universal conductance fluctuations (UCF, right panels) as a function of the displacement of both leads from their symmetry-respecting positions for systems with fixed internal symmetry. The displacement is measured in terms of $\lambda = 1 - W_\Gamma/W$. The data points (circles with a variety of filling styles) are obtained from an average over 5000 realizations of the RMT model described in the text ($M=1000$, $N=50$). The curves show semiclassical prediction (41) for WL and Eq. (42) for UCF. Labels “ $A \rightarrow B$ ” specify the symmetry of the lead arrangement at $\lambda=0$ (symmetric arrangement) and $\lambda=1$ (where at least one of the symmetries is fully removed). In these labels, the subscript ₄ on A or B indicates that the internal symmetry is fourfold; if this subscript is not present the internal symmetry is identical to the one specified by A . In this figure, the magnetic field is set to $B=0$.

extends to the symmetry breaking in the internal dynamics, which then requires to interpolate between ensembles of Table I. Following earlier works, the RMT model can be further utilized to include the effects of dephasing and a finite Ehrenfest time. For dephasing, this is achieved by opening additional ports which couple to a voltage probe [25] or

a dephasing stub [26]. A finite Ehrenfest time is obtained when F represents a dynamical system such as the kicked rotator [11] (which also possesses discrete symmetries). This strategy can also be used to probe the case of dynamics which are not fully chaotic (which in the kicked rotator is achieved for moderate values of the kicking strength).

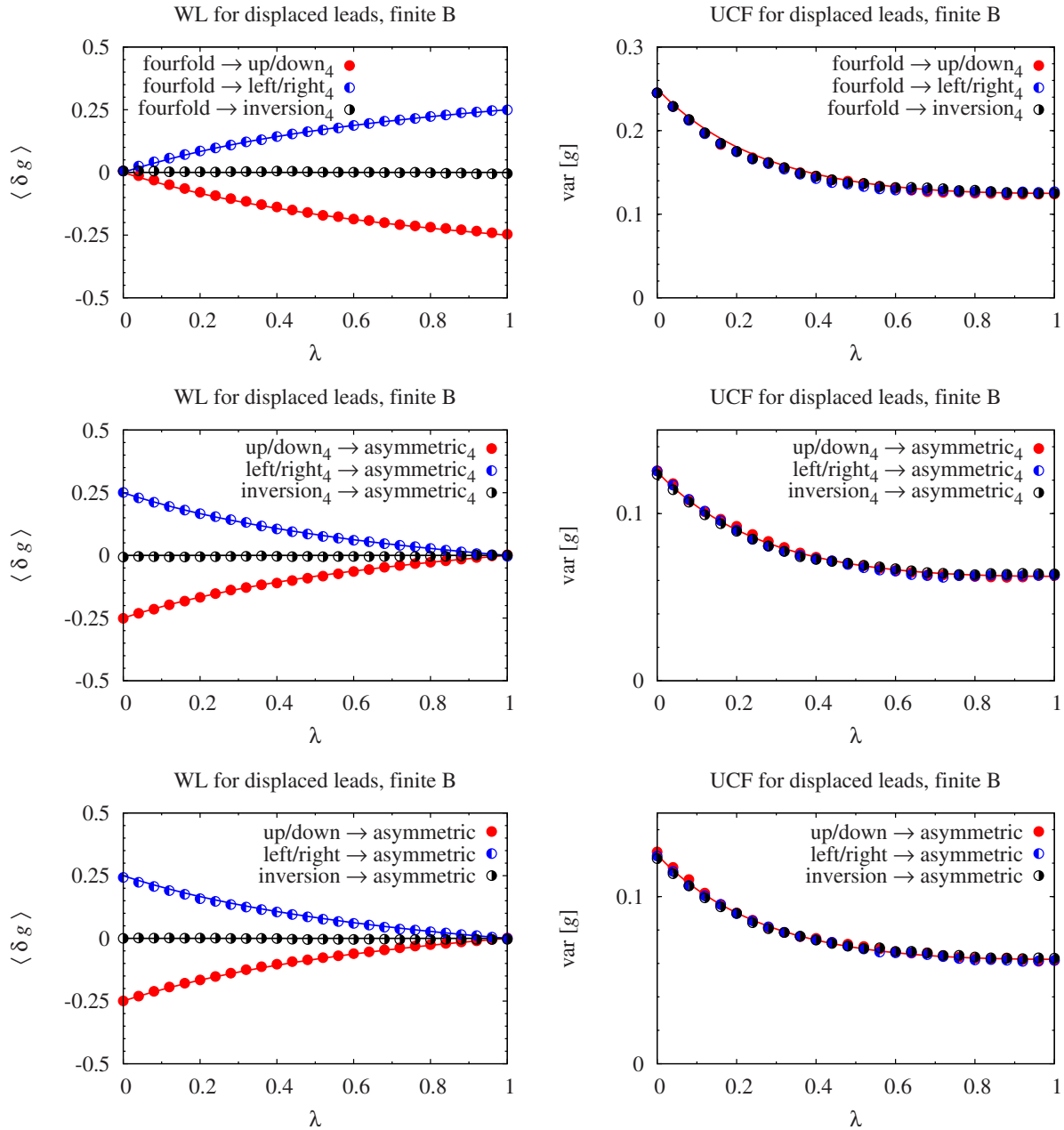


FIG. 8. (Color online) Same as Fig. 7 but for a finite magnetic field.

IX. CONCLUDING REMARKS

The transport calculations performed here assume that the classical dynamics is uniformly chaotic and in particular do not apply to system with islands of stability in phase space (such as the annular billiard studied in Refs. [27,28]) or networks of chaotic dots interconnected by narrow leads (such as the double dot in Ref. [7]). It would be intriguing to study the shape of the backscattering and forward-scattering peaks for such systems.

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APPENDIX: OBTAINING UCFS FROM THE VARIANCE OF REFLECTION

In Sec. VII we pointed out that unitarity implies $\text{covar}[R_L, R_R] = \text{var}[R_L] = \text{var}[R_R]$. Here we outline a semiclassical calculation of $\text{var}[R_L]$, which acts as a check of the semiclassical calculation of $\text{covar}[R_L, R_R]$ in Sec. VII. The rules to calculate each contribution remain the same as for $\text{covar}[R_L, R_R]$. However, the contributions that we consider differ by the requirement that all paths start and end on the same lead L.

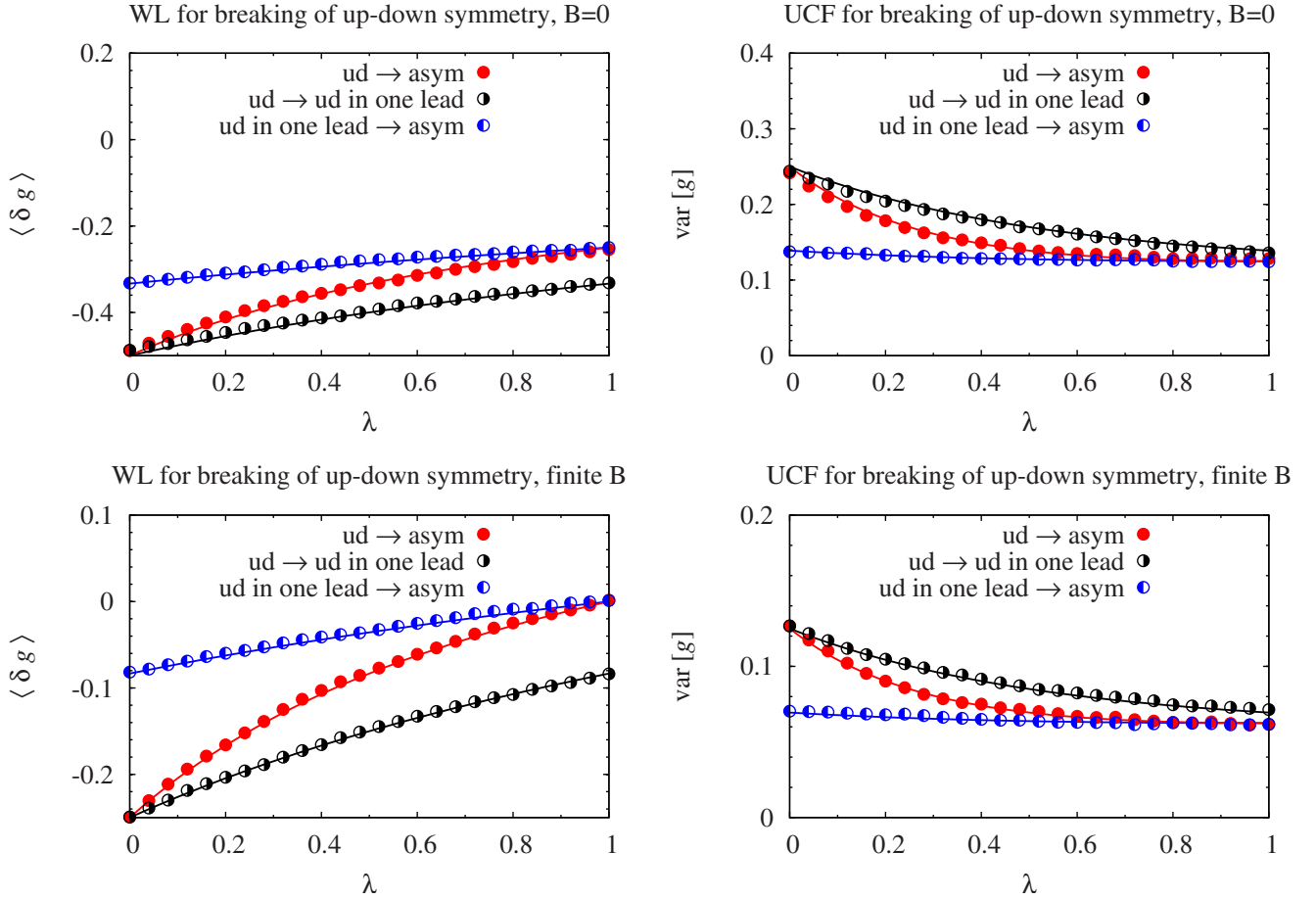


FIG. 9. (Color online) Same as Figs. 7 and 8, but comparing the displacement of both leads for internal up-down symmetry (solid circles) to the displacement of the first lead (circles filled on the right), followed by the displacement of the second lead (circles filled on the left).

We know that the result must be invariant under the interchange of labels “L” and “R” and this invariance is manifestly obvious in the contributions to $\text{covar}[R_L, R_R]$. In contrast, this invariance is hidden in the contributions to $\text{var}[R_L]$ that we discuss here. Thus, the simplest check that one has not missed any contributions is that this invariance is present when one sums the contributions.

1. Up-down symmetric dot

In the case of an up-down symmetric dot, all contributions in both Figs. 4 and 5 contribute to $\text{var}[R_L]$ once we change all lead labels so that “R” \rightarrow “L” and “ \cap R” \rightarrow “ \cap L” (but not vice versa). Writing contributions to $\text{var}[R_L]$ with a “prime” (to distinguish them from contributions to $\text{covar}[R_L, R_R]$) we find

$$C'_v + C'_{vi} + C'_i = \frac{2 N_{\cap L}^2 (N_L + N_R)^2 + 4 N_{\cap L} (N_L - N_{\cap L}) N_L (N_L + N_R) + 4 (N_L - N_{\cap L})^2 N_L^2}{\beta (2N_L + 2N_R - N_{\cap L} - N_{\cap R})^2 (N_L + N_R)^2}, \quad (\text{A1})$$

$$C'_{vii} + C'_{viii} + C'_{ii} + C'_{iii} = -\frac{2 N_L^2 [N_{\cap L} (N_L + N_R) + 2 (N_L - N_{\cap L}) N_L]}{\beta (2N_L + 2N_R - N_{\cap L} - N_{\cap R}) (N_L + N_R)^3}, \quad (\text{A2})$$

$$C'_{iv} = \frac{2 N_L^4}{\beta (N_L + N_R)^4}. \quad (\text{A3})$$

As in Sec. VII, we find that this sum is most easily evaluated by rewriting the contributions in terms $n_\kappa = N_\kappa / (N_L + N_R)$ and $w_\kappa = 1 - N_{\cap\kappa} / N_\kappa$ for $\kappa = L, R$. Performing a little algebra using $n_L + n_R = 1$, we then recover Eq. (32) and, therefore, $\text{var}[R_L] = \text{covar}[R_L, R_R]$. Furthermore, Eq. (32) is invariant under the interchange of labels L and R, which entails $\text{var}[R_R] = \text{var}[R_L]$. Thus, the semiclassical method obeys the relations $\text{var}[R_L] = \text{var}[R_R] = \text{covar}[R_L, R_R]$ as required by the unitarity of the scattering matrix. This strongly suggests that we have not missed any contributions and gives us confidence in the result; particularly, it is noteworthy that the individual contributions in $\text{var}[R_L]$ and $\text{covar}[R_L, R_R]$ combine in very different ways to give the invariance under the interchange of L and R.

2. Left-right or inversion-symmetric dot

The evaluation of $\text{var}[R_L]$ for a left-right or inversion-symmetric dot is very similar to that for an up-down symmetric dot. However, here, when a path hits the L lead then

its image hits the R lead. This means that there are no contributions to $\text{var}[R_L]$ of the form shown in Fig. 5, since all paths must go from the L lead to the L lead. Thus, to get $\text{var}[R_L]$ for a left-right or inversion-symmetric dot, we need to subtract those contributions from the result for $\text{var}[R_L]$ in an up-down symmetric dot. The sum of these contributions to $\text{var}[R_L]$, written with the same denominator as in Eq. (33), is

$$C'_v + C'_{vi} + C'_{vii} + C'_{viii} = \frac{2}{\beta} \frac{N_\cap^2 (N_L^2 - N_R^2)^2}{(2N_L + N_R - 2N_\cap)^2 (N_L + N_R)^4}, \quad (\text{A4})$$

This only differs by an overall sign from the sum of contributions in Eq. (36). Subtracting this from the result Eq. (33), we get $\text{var}[R_L]$ for a left-right or inversion-symmetric dot. The result equals $\text{covar}[R_L, R_R]$ given by Eq. (37); thus, we have $\text{covar}[R_L, R_R] = \text{var}[R_L] = \text{var}[R_R]$ as required by the unitarity of the scattering matrix.

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